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GAMES PLAYED THROUGH AGENTS

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Games Played through Agents *

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Abstract

We introduce a game of complete information with multiple principals and multiple agents. Each agent takes an action which can affect the payoffs of all principals and all agents. Each principal offers monetary transfers to each agent conditional on the action taken by the agent. We characterize pure-strategy equilibria and we provide conditions – in terms of game balancedness – for the existence of an equilibrium with an efficient outcome. Games played through agents display a type of strategic inefficiency which is absent when either there is a unique principal or there is a unique agent.

1 Introduction

A game played though agents occurs when a set of players (*the agents*) take decisions that affect the payoffs of another set of players (*the principals*) and the principals can, by means of monetary inducements, try to influence the decisions of the agents. In other words, a game played though agents is a multi-principal multi-agent game.

The original principal-agent framework – which has one principal and one agent – has been extended in a general way in two directions: (1) Many principals and one agent (Bernheim and Whinston's [3] common agency); and (2) One principal and many agents (Segal's [12] contracting with externalities). The objective of this paper is to consider the general case with many principals and many agents. Multi-principal multi-agent problems arise in political economy, industrial organization, and auction theory:

Lobbying A widespread way of modeling interest group politics is through common agency (e.g. Dixit, Grossman, and Helpman [6]). There are many lobbies (principals) and one politician (the agent). However, modern democracies are characterized by a multiplicity of public decision-makers. This is true both in terms of organs (division of powers) and in terms of organ members (many organs – such as parlaments – are collegial). Interest groups with opposing interests will fight eachother by trying to manoeuver the decision-making of several

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agents. Hence, it would be important to know if there are differences between the case with one politician and the case with multiple politicians.[1]

Supply Contracts An industry with several firms (*retailers*) uses inputs produced by another industry with several firms (*suppliers*). The suppliers propose contracts to the retailers. A contract proposed by one supplier may cover not only the relation between that supplier and the retailer, but also the relation between the retailer ans the other suppliers (such as an exclusionary clause). The problem with several suppliers but only one retailer has been studied in a general way by Bernheim and Whinston [4], with the important conclusion that, without restrictions on bilateral retailer-manufaturing contracting, the contractual arrangements that arise in equilibrium are efficient (Bork's Thesis). Does this efficiency result hold when there are multiple retailers?

Auctions Common agency can be regarded as a generalized form of auction, in which the auctioneer has preferences over the final allocation, bidders may have externalities with eachother, and multiple units may be sold (See Bernheim and Whinston [3]). The buyers are the principals and the auctioneer is the agent. However, common agency assumes that the auctioneer is unique. In reality there may be several auctions with the same buyers but different auctioneers. In the wave of privatizations that has swept Europe, the same buyers (or subsets thereof) have faced eachother in asset sales run by different auctioneers (national governments). The problem is made interesting by the fact that the buyers' payoffs are not separable in the outcome of the different auctions. For instance, economies of scale may lead to conjecture that the payoff from winning both auctions in two neighboring countries is higher than the sum of the payoffs of winning the auction in each country separately. A common agency auction considered in isolation always has an efficient equilibrium (in which the allocation maximizes the sum of bidders' valuations). Is it true that several common agency auctions viewed together always have an efficient equilibrium?[2]

A game played through agents is as follows. There are a set of principals and a set of agents. Each agent must choose an action out of a feasible set of actions (policy choices in the case of lobbying, quantity orders in supply contracts, or object allocations in auctions). Each principal offers to each agent a schedule of monetary transfers contingent on the agent choosing a certain action (campaign contributions, supply contracts, or bids). Given the principals's transfer schedules, an agent chooses his action to maximize the sum of transfers he receives from the principals minus the cost of undertaking the action. A principal chooses his transfer schedules to maximize the utility from the agents' actions minus the sum of transfers he makes to agents.

So far, we have been intentionally vague about timing. The simplest structure is a two-stage game, in which first all principals simultaneously choose their transfer schedules and then all agents observe the schedules and simultaneously choose their actions. We will work with this timing structure for most of the paper. However, some of the examples above present more complicated timing structures, which include elements of sequentiality. For instance,

[1] Groseclose and Snyder's [7] constitute exceptions in that they consider multiple policy-makers. In Section 4, we will consider Groseclose and Snyder's vote buying model in detail.

[2] Another interesting example of games played through agents is in international taxation: national governments (principals) compete to attract international firms (agents) by offering subsidies and tax breaks to firms that relocate on their territory. See Besley and Seabright [5].

it would be a coincidence that two countries hold auctions simultaneously. More likely, one auction starts after the outcome of the other is known. In Section 8 we will consider possible sequential variants of the simultaneous game and show that the main results still hold.

Our main focus is efficiency, which we define according to the Utilitarian criterion. An outcome is efficient if it maximizes the sum of the payoffs of all agents and all principals. If there is a unique agent, Bernheim and Whinston have shown that there always exist an equilibrium (the *truthful equilibrium*) that produces an efficient outcome. If, instead, there is a unique principal, Segal shows that, if a certain type of externalities among agents' payoff functions is absent, then there always exists an efficient equilibrium. Hence, in both these limit cases, if there are no direct externalities among agents, efficient equilibria exist.

However, it turns out that, even when there are no direct externalities among agents, a multi-principal multi-agent game need not have an efficient equilibrium. The presence of multiple players on both sides creates a strategic externality that makes it impossible to achieve the efficient outcome. The main result of this paper is to provide a general necessary and sufficient condition for the existence of an efficient equilibrium. This condition relates to the cooperative concept of balancedness, which we extend – with some important differences – to our game. In the present context, balancedness has a noncooperative interpretation in terms of weighted deviations from the equilibrium outcome and sheds light on the nature of the strategic interaction between principals and agents. Moreover, the balancedness of a game can be checked in a straightforward way.

The organization of the paper is as follows. As the existence of inefficiency in our model does not depend on direct externalities among agents, we develop our core argument under the assumption that agents have no direct preference on outcomes but only want to maximize the sum of transfers they receive from principals. This keeps notation lighter and allows to focus on the original contribution of the paper. Only in the end of the paper, we consider the general case in which agents have also direct preferences and we show that our results extend.

We begin with the formal presentation of the game in Section 2. In Section 3 we give a characterization of pure-strategy equilibria that we will use in the rest of the paper. In Section 4, we focus on a simplified version of the game, in which each agent has only two possible actions. This simplification avoids issues of coordination among principals. The main result is a necessary and sufficient condition for the existence of an efficient equilibrium, which we then discuss in relation of to the literature. In Section 5 we allow agents to have more than two actions. To deal with coordination problems, we introduce and study *weakly truthful equilibria*, which are an extension of Bernheim and Whinston's truthful equilibrium to games with many agents. We give necessary and sufficient conditions for their existence. In Section 6, we extend our analysis to the case in which agents have direct preferences on outcomes. In Section 7 we show that a mixed-strategy equilibrium must always exist. In Section 8, we probe the robustness of our results against alternative timing structures, in which either principals move sequentially (*principal sequentiality*) or agents move sequentially (*agent sequentiality*). Section 9 concludes by examining the implications of our results for the three examples of games played through agents discussed above.

2 The Game

There is a set M of principals and a set N of agents. Let m denote the typical element of M and n the typical element of N. We emphasize that there is no natural relation between any of the principals and any of the agents. The game takes place in two stages: first the principals move simultaneously, then the agents move simultaneously.[3]

Each agent has a finite pure set of *actions* S^n. Let $S \equiv \prod_{n\in N} S^n$ and $H = \cup_{n\in N} S^N$. The typical element of S is an outcome $s = (s^1, \dots, s^n)$ and the typical element of H is a pair (n, s^n). Each principal chooses a vector of nonnegative *transfers* $t^m \in \Re^{+H}$ which specifies the transfer from her to each agent for each action of that agent. Thus, $t^{mn}_{s^n}$ is the tranfer of principal m to agent n conditional on agent n choosing action s^n. Agent n receives money only for the action that he actually chooses, but he may receive money from more than one principal.

For most of the paper, we assume that agents only care about the amount of monetary transfers they collect, and not about the actions that they take. Agent n's payoff if he chooses action s^n is $\sum_{m\in M} t^{mn}_{s^n}$. Principals care both about money and the actions that agents choose. Let G^m_s be the gross payoff to Principal m if action s is chosen by the agents. The net payoff of principals is assumed to be separable in gross payoff and money. The net payoff to Principal m if she offers transfers $\{t^{mn}_{s^n}\}_{(n,s^n)\in H}$ and agents choose $\hat{s}$ is $G^m_{\hat{s}} - \sum_{n\in N} t^{mn}_{\hat{s}^n}$.[4]

The extensive form game is as follows. First, each principal chooses her vector of transfers to the agents simultaneously and noncooperatively. Second, the vectors of all principals are publicly announced to agents, who then choose their actions.

We focus here on pure strategies.[5] The strategy set of Principal m is the subset $T^m \equiv \Re^{+H}$. A pure strategy for m is simply an element of T^m. Let $T \equiv \Pi_{m\in M} T^m$. The strategy set for Agent n is S^n. A pure strategy for Agent n is $\sigma^n : T \to S^n$. A *pure-strategy equilibrium* of the transfer game is a subgame-perfect equilibrium of the two-stage game in which each agent and each principals uses a pure-strategy:

Definition 1 *A pure strategy equilibrium of a transfer game is a pair $(\hat{t}, \hat{\sigma})$, where $\hat{t} = (\hat{t}^{mn}_{s^n})_{m\in M, n\in N, s^n\in S^n}$ and $\hat{\sigma} = (\hat{\sigma}^n)_{n\in N}$, in which:*

(i) For every $n \in N$, and every $t \in T$,

$$\hat{\sigma}^n(t) \in \ argmax_{\ s^n\in S^n} \sum_{m\in M} t^{mn}_{s^n}$$

(ii) For every $m \in M$, given $(\hat{t}^j)_{j\neq m}$, $\hat{t}^m$ solves:

$$\max_{t^m} G^m_s - \sum_{n\in N} t^{mn}_{s^n}$$

subject to

$$s^n = \hat{\sigma}(t^m, \hat{t}^{-m}).$$

We now define efficiency. Following Bernheim and Whinston [3] and Segal [12], we use a utilitarian criterion. An action is efficient if it maximizes the sum of the net payoffs of all players (agents and principals). Transfers can be neglected, and the definition of efficiency is:

[3]Sequential variations of the game are considered in Section 8.

[4]The separability assumption does not appear to be crucial to the results presented here, as it is not crucial to the results obtained in common agency (Dixit, Grossman, and Helpman [6]).

[5]Mixed strategies are considered in Section 7.

Definition 2 *An action s^* is efficient if*

$$\sum_{m\in M} G^m_{s^*} + \geq \sum_{m\in M} G^m_s \tag{1}$$

for every $s \in S$.

The *outcome* of an equilibrium is the action profile chosen by agent in that equilibrium. We will sometimes say that an "equilibrium is efficient," meaning that the outcome of that equilibrium is efficient.

3 A First Characterization of Pure Strategy Equilibria

A pure strategy equilibrium is characterized by three conditions, which are formally reported in Theorem 1 below. They are derived using the idea, common in principal-agent problems, that we may think of principals as choosing the action of the agents, provided they give the appropriate incentive to the agents. The conditions are:

1. Each agent chooses an action that maximizes his payoff. This is the condition (AM) (Agent Maximization) below.

2. Given the transfers of the other principals, Principal m can induce agents to choose any particular action provided that she puts high enough transfers on that action. The cost for m to induce s rather than $\hat{s}$ is $\sum_{j\neq m} \hat{t}^{jn}_{s^n} - \sum_{j\neq m} \hat{t}^{jn}_{\hat{s}^n}$, which is the minimum sum of transfers sufficient to induce agents to move from $\hat{s}$ to s. The benefit is $G^m_s - G^m_{\hat{s}}$. If $\hat{s}$ is an equilibrium, then the cost of a deviation must be greater than the benefit of a deviation for each m and each s, which is what Condition (IC) (Incentive Compatibility) requires.

3. Each principal sets his transfers so that the cost of implementing $\hat{s}$ is minimal. There cannot be a way in which principal m reduces her equilibrium transfers for $\hat{s}$ without deviating from $\hat{s}$. This is condition (CM) (Cost Minimization).[6]

Formally, this is the characterization.

Theorem 1 *A pair $(\hat{t}, \hat{s})$ of transfers and action profiles is a pure strategy equilibrium outcome if and only if the following conditions are satisfied:*

(AM) for every $n \in N$, $s^n \in S^n$,

$$\sum_{m\in M} \hat{t}^{mn}_{\hat{s}^n} \geq \sum_{m\in M} \hat{t}^{mn}_{s^n};$$

(IC) for every $m \in M$, $s \in S$,

$$G^m_{\hat{s}} + \sum_{n\in N} \sum_{j\neq m} \hat{t}^{jn}_{\hat{s}^n} \geq G^m_s + \sum_{n\in N} \sum_{j\neq m} \hat{t}^{jn}_{s^n};$$

[6]Note that what we call (AM) is what is usually called incentive-compatibility in principal-agent problems. However, it is useful here to use the term incentive compatibility for the principals' choices. While the agent maximization problem is characterized by one condition (AM), the principal maximization problem is characterized by two conditions (IC) and (CM). Then it is useful to distinguish between principal incentive compatibility (which is across actions) and principal cost minimization (which is for the equilibrium action).

(CM) for every $m \in M$, $n \in N$,

$$\sum_{m \in M} \hat{t}^{mn}_{\hat{s}^n} = \max_{s^n \in S^n} \sum_{j \neq m} \hat{t}^{jn}_{s^n}.$$

Proof: Condition (AM) is clearly necessary and sufficient for the action $\hat{s}^n$ to be a best response of the agent n to the transfers of the principals.

To prove the statement for the two remaining conditions, we characterize the best response of a principal m to a given choice $(t^j)_{j \neq m}$ of transfers of the other principals. To lighten the notation, we write:

$$T^{mn}_{s^n} \equiv \sum_{j \neq m} \hat{t}^{jm}_{s^n}.$$

Clearly for m the only relevant quantity is value of the matrix $(T^{mn}_{s^n})_{n \in N, s^n \in S^n}$. Consider now the maximum over the agents' actions of the transfers that the other principals are promising to him, and the direct payoff of the agents: $\max_{a \in S^n} T^{mn}_a$.

The principal m can induce from the agents the choice of any vector of actions $s \in S$ provided he promises a transfer greater than

$$(\max_{a \in S^n} T^{mn}_a) - T^{mn}_{s^n}.$$

to the agent n for the action s^n. Principal m will not choose $\hat{s}$ (and $(\hat{s}, \hat{t})$ is not an equilibrium) unless $\hat{s}$ solves

$$\max_{s \in S} G^m_s - \sum_{n \in N} [(\max_{a \in S^n} T^{mn}_a) - T^{mn}_{s^n}]. \tag{2}$$

But $\sum_{n \in N} \max_{a \in S^n} T^{mn}_a$ is a constant independent of s, so $\hat{s}$ solves the problem (2) if and only if it satisfies (IC).

Finally we consider the condition (CM). The following lemma is very simple, but we state it for convenience. The proof is elementary.

Lemma 1 *For every action profile $\tilde{s}$ and every vector T^m, the cost minimization problem in $t^m = (t^{mn}_{s^n})_{n \in N, s \in S} \geq 0$*

$$\min_{t^m} \sum_{n \in N} t^{mn}_{\tilde{s}^n} \text{ subject to } t^{mn}_{\tilde{s}^n} + T^{mn}_{\tilde{s}^n} \geq t^{mn}_{s^n} + T^{mn}_{s^n}, \text{ for every } n \in N, s \in S \tag{3}$$

has value $c(\tilde{s}, T^m)$ equal to $\sum_{n \in N} [(\max_{a \in S^n} T^{mn}_a) - T^{mn}_{\tilde{s}^n}]$, and solution any $t^{mn}_{s^n} \geq 0$ such that:

$$t^{mn}_{\tilde{s}^n} = (\max_{a \in S^n} T^{mn}_a) - T^{mn}_{\tilde{s}^n},$$

$$t^{mn}_{s^n} \leq (\max_{a \in S^n} T^{mn}_a) - T^{mn}_{s^n}, \text{ for every } s^n.$$

If we apply the lemma choosing as $\tilde{s}$ the candidate equilibrium action profile $\hat{s}$, we get that t^m is a solution of the problem of minimum cost to implement $\hat{s}$ if and only if:

$$\hat{t}^{mn}_{\hat{s}^n} = \max_{a \in S^n} T^{mn}_a - T^{mn}_{\hat{s}^n} \tag{4}$$

and

$$\hat{t}^{mn}_{s^n} \leq \max_{a \in S^n} T^{mn}_a - T^{mn}_{s^n} \tag{5}$$

for every s. But (4), (5), and (AM) are equivalent to (4) and (AM). Since (4) is (CM), we have concluded our proof. ∎

Some of the properties of an equilibrium are worth pointing out explicitly:

Corollary 1 *for all n there is an $\tilde{s}^n \in S^n \setminus \hat{s}^n$ such that:*

$$\sum_{m \in M} t^{mn}_{\hat{s}^n} = \sum_{m \in M} t^{mn}_{\tilde{s}^n}. \tag{6}$$

Proof: Suppose for some n $\sum_{m \in M} t^{mn}_{\hat{s}^n} > \sum_{m \in M} t^{mn}_{\tilde{s}^n}$ (the case "<" is prevented by (AM)). Take any m. By (CM), either $\hat{t}^{mn}_{s^n} = 0$ or there exists a $\tilde{s}$ different from $\hat{s}$ such that $\sum_{j \neq m} t^{mn}_{\tilde{s}^n} = \sum_{m \in M} t^{mn}_{\hat{s}^n}$. The second case would be an immediate contradiction. Suppose then that $\hat{t}^{mn}_{s^n} = 0$. This is true for all m. Hence, $\sum_{m \in M} t^{mn}_{\hat{s}^n} = 0$ and we have a contradiction. ∎

For every agent, there exists an action different from the equilibrium action with the same amount of transfers of the equilibrium action. This is an immediate consequence of (CM). If this were not the case, then at least one of the principals could reduce her transfer on $\hat{t}$.

Moreover, for every n and every m, there exists an action $a(m,n)$ (which could be $\hat{s}^n$) such that

$$\text{(i) } \sum_{j \in M} t^{jn}_{a} = \sum_{j \in M} t^{jn}_{\hat{s}^n}, \text{ and (ii) } t^{mn}_{a} = 0. \tag{7}$$

Consider the subset of S^n for which Agent n gets the maximal amount of transfers (and we know from Corollary 1 that this subset has at least two elements). (7) says that each principal makes a zero transfer on at least one of the actions in this subset. If this were not true, (CM) would be violated. Principal m could reduce her equilibrium transfers without modifying the outcome. Equation 7 implies that, given the equilibrium transfers, if a principal disappeared (and all her transfers were zero), none of the agents would be hurt.

4 Agents with Two Actions

In this section we introduce the main results of the paper in a simplified environment in which each agent has only two actions. We proceed as follows: Subsection 4.1 restates the characterization theorem in this simplifies environment. Subsection 4.2 provides four examples. Subsection 4.3 states the main theorem: a necessary and sufficient condition for the existence of an efficient equilibrium. Subsection 4.4 discusses the condition.

4.1 Characterization

We denote by $(s^n)'$ the action of n different from s^n. The following is the equivalent of Theorem 1 in this simplified environment:

Proposition 1 *If $\sharp S^n = 2$ for every $n \in N$, then the pair $(\hat{t}, \hat{s})$ is a pure strategy equilibrium outcome if and only if*

(AM) For every $n \in N$, $s \in S$

$$\sum_{m \in M} \hat{t}^{mn}_{\hat{s}^n} = \sum_{m \in M} \hat{t}^{mn}_{(\hat{s}^n)'};$$

(IC) For every $m \in M$, $s \in S$:

$$G^m_{\hat{s}} + \sum_{n \in N} \sum_{j \neq m} \hat{t}^{jn}_{\hat{s}^n} \geq G^m_s + \sum_{n \in N} \sum_{j \neq m} \hat{t}^{jn}_{\hat{s}^n};$$

(CM) For every $m \in M$, $n \in N$,

$$\text{if } \hat{t}^{mn}_{\hat{s}^n} > 0 \text{ then } \hat{t}^{mn}_{(s^n)'} = 0.$$

Proof: Immediate from Theorem 1 and (6). ∎

With two actions per agent, (CM) implies that, for each agent, no principal can make more than one strictly positive transfer the sum of transfers for one action is exactly equal to the sum of transfers for the other action. If either of these conditions is violated, some principal can reduce their transfers without changing the outcome.

A couple of observations are straightforward from Proposition 1. By summing (AM) over n and subtracting it from (IC) applied to $s = ((\hat{s}^n)', \hat{s}^{-n})$, we have that for all n

$$G_{\hat{s}} - G^m_{((\hat{s}^n)', \hat{s}^{-n})} \geq \hat{t}^{mn}_{\hat{s}^n} - \hat{t}^{mn}_{(\hat{s}^n)'}$$

which, combined with (CM) implies that for any m and any n

$$\begin{array}{ll} \text{either} & \hat{t}^{mn}_{\hat{s}^n} = 0 \text{ and } \hat{t}^{mn}_{(\hat{s}^n)'} \geq G^m_{((\hat{s}^n)', \hat{s}^{-n})} - G^m_{\hat{s}} \\ \text{or} & \hat{t}^{mn}_{\hat{s}^n} \leq G^m_{\hat{s}} - G^m_{((\hat{s}^n)', \hat{s}^{-n})} \text{ and } \hat{t}^{mn}_{(\hat{s}^n)'} = 0. \end{array} \tag{8}$$

Moreover, in the two actions case, the action outcome of a pure strategy equilibrium is efficient:

Proposition 2 *If $\sharp S^n = 2$ for every $n \in N$, then pure strategy equilibria are efficient.*

Proof: If we add (IC) over $m \in M$ we get that for every s:

$$\sum_{m \in M} G^m_{\hat{s}} + (M-1) \sum_{n \in N} \sum_{m \in M} \hat{t}^{jm}_{\hat{s}^n} \geq \sum_{m \in M} G^m_s + (M-1) \sum_{n \in N} \sum_{m \in M} \hat{t}^{jm}_{s^n},$$

which, by (AM), implies $\sum_{m \in M} G^m_{\hat{s}} \geq \sum_{m \in M} G^m_s$. ∎

The assumption that $\sharp S^n = 2$ is essential. As we shall see in Section 5, with more than three actions a pure strategy equilibrium need not be efficient. Instead, the assumption that agents do not care about actions is not essential. Proposition 2 can easily be proved in the case in which agents care about actions.

4.2 Examples

We consider a few examples with $M = N = \{1, 2\}$, $F^n \equiv 0$ and $\sharp S^i = 2$ for both agents. We denote the agents' actions as $S^1 = \{T, B\}$ and $S^2 = \{L, R\}$. We adopt the convention of presenting the payoff matrix in the form:

	L	R
T	G^1_{TL}, G^2_{TL}	G^1_{TR}, G^2_{TR}
B	G^1_{BL}, G^2_{BL}	G^1_{BR}, G^2_{BR}

It is important to keep in mind that this is not the usual payoff matrix. The actions refer to agents while the payoffs refer to principals. Also, these are gross payoffs. The net payoffs will be given by the gross payoffs minus the transfers. The transfer vector t^m is written as $(t_T^{m1}, t_B^{m1}; t_L^{m2}, t_R^{m2})$.

Prisoner's Dilemma The payoffs of the principals are:

	L	R
T	x, x	z, y
B	y, z	$0, 0$

with $y > x > 0 > z$ and $2x > y + z$. The efficient action is unique: (T, L). Hence, by Proposition 2, if a pure-strategy equilibrium exists, it must have (T, L) as outcome. By (CM), the first principal does not pay for the action T, and the second does not pay for L. By (AM), the payment on each action from the two principals must be the same; so the equilibrium transfers are pairs of the form:

$$t^1 = (0, a; b, 0), t^2 = (a, 0; 0, b). \tag{9}$$

The (IC) condition for the first principal is $x + a \geq \max\{z + a + b, y, b\}$, and $x + b \geq \max\{z + a + b, y, a\}$ for the second. So the set of pure strategy equilibria is given by any transfer with (a, b) such that $a, b \in [y - x, x - z]$ *and* $x \geq b - a \geq -x$. In particular, there exists a minimal transfer equilibrium in which $a = b = y - x$. The agents choose T, respectively L, whenever indifferent. The rent of each agent is at least the difference between the best outcome and the "cooperate" outcome, but can get as high as the difference between "cooperate" and the bad outcome.

Coordination Game The payoffs of the principals are:

	L	R
T	x^1, y^1	$0, 0$
B	$0, 0$	x^2, y^2

with

$$x^1 + y^1 > x^2 + y^2, y^1 \leq y^2, x^i, y^i \geq 0, i = 1, 2$$

Again there is a unique efficient outcome, (T, L), hence a unique equilibrium outcome in pure strategy, if any. From Proposition 1 it is easy to see that there exists an equilibrium if there are transfers $t^1 = (a, 0; b, 0)$, $t^2 = (0, a; 0, b)$ that satisfy

$$x^1 - x^2 \geq a + b \geq y^2 - y^1, \tag{10}$$

Clearly, (10) is satisfied for the parameters under consideration and an equilibrium with outcome (T, L) always exists. The combined rent of the two agents is at least $y^2 - y^1$.

In the extreme "pure", coordination game with $x^1 = y^1 > x^2 = y^2$, zero transfers from both principals, for each agent and each action is an equilibrium.

The prisoners' dilemma and the coordination game, when played through agents, have a unique pure-strategy equilibrium, and this equilibrium is efficient. The following examples instead show that there are games in which a pure-strategy equilibrium does not exist and all other equilibria are inefficient.

Opposite Interests Game

	L	R
T	$3,0$	$0,x$
B	$0,x$	$0,x$

with $1.5 < x < 3$. The only possible pure strategy equilibrium outcome is (T, L), with transfers $t^1 = (a, 0; b, 0)$ and $t^2 = (0, a; 0, b)$. The (IC) for the two principals are, respectively:

$$3 \geq \max\{a, b, a + b\},$$

$$a + b \geq \max\{x + a, x + b, x\}.$$

Together, they imply $3 \geq a + b \geq 2x$, which cannot be satisfied if $x > 1.5$. No pure strategy equilibrium exists and all other equilibria are necessarily inefficient because they involve outcomes different from (T, L) with positive probability.

This game can be interpreted as a lobbying problem. Principal 1 is a lobby who wants to change the status quo. Principal 2 wants to keep things as they are. In order to change the status quo, Principal 1 needs approval from two governmental bodies, Agent 1 and Agent 2. The efficient outcome is to change the status quo. However, Prncipal 2 enjoys a strategic advantage because he only needs to convince one of the two agents to say no.

With some re-working, the Opposirte Interest Game can also be interpreted as a very basic supply contract problem with two manufaturers (principals) and two retailers (agents). Retailers are in separate markets (and therefore do not impose externalities on each other). Each retailer needs exactly one unit of the input good produced by the manufacturers. The total cost functions of the two manufactureres are as follows:

q	0	1	2
C^1	0	3	3
C^2	0	1	4

Manufacturer 1 has economies of scale and Manufacturer 2 has diseconomies. The efficient allocation would be that 1 produces two units and 2 produces nothing. Let T represent Retailer 1 buying his unit from Manufaturer 1 and let L represent Retailer 2 buying his unit from Manufaturer 2. B and R are the opposite actions. Suppose that there is a 'fixed' price of 3 per unit but manufacturers can offer discounts (this is a quick way to overcome the nonnegativity constraint – the whole analysis of this paper can be re-done without the nonnegativity constraint or with other constraints). For instance, t_L^{12} is the discount over the fixed price of 3 that Principal 1 offers to Agent 1 if he buys from her. Then, it is easy to check that this supply contract problem is exactly equivalent to the Opposite Interest Game examined above and has no efficient equilibrium. In order to achieve efficiency, Principal 1 should sell to both retailers but Principal 2 can easily undercut her on one of the two retailers. The noncontractible externality here is that, if Principal 2 sells to Retailer 2, there is an increase in the cost of production for the good that Principal 1 is still selling to Agent 1.[7]

[7] In Subsection 8.2 we will re-interpret the agent-sequential version of the Opposite Interest Game as a sequence of two auctions.

Voting Game Our last example has more than 2 agents and is inspired by Groseclose and Snyder [7]. There are two principals, $M = \{1, 2\}$, and an odd number $N = 2K + 1$ of agents. Each agent may vote for one of two alternatives, also labelled 1 and 2 and he may not abstain. The alternative with the larger number of votes is chosen. The payoff of the principal 1 is $x > 1$ if the alternative 1 is chosen, and 0 if 2 is chosen. The payoff of Principal 2 is 1 if 2 is chosen and 0 otherwise. Thus, all action profiles such that $\sharp\{n \in N | s^n = 1\} \geq K + 1$ are efficient.

This game has no equilibrium in which alternative 1 is chosen for sure, and hence it has only inefficient equilibria. To see this, suppose that an equilibrium exists, where alternative 1 is chosen for sure. In this equilibrium, Principal 2 must be paying no money to agents. If it were not so, Principal 2 would get a negative payoff while she can always ensure a zero payoff by offering zero to all agents. There are two cases: (i) Principal 1 makes a strictly positive offer for certain to all agents; (ii) There is an agent n that receives zero offers from both Principal 1 and Principal 2. In case (i), given any strategy of Principal 2, Principal 1 can still guarantee herself Alternative 1 but save money by making a zero offer to one of the agents. In case (ii) Principal 1 could offer a zero transfer to one of the agents she is currently offering a strictly positive transfer and make an infinitesimal transfer to the agent who is not receiving anything. This shows that no equilibrium in which Alternative 1 is chosen for sure exists.[8]

4.3 Existence of pure strategy equilibria: Necessary and sufficient condition

In this section we provide a necessary and sufficient condition for the existence of a pure-strategy equilibrium. We know that pure-strategy equilibria are efficient (Proposition 2). It is also easy to see that if an efficient mixed-strategy equilibrium exists, then there must also exist an efficient pure-strategy equilibrium. Hence, for a game played through agents in which agents have only two actions existence of pure-strategy equilibria and existence of efficient equilibria are equivalent.[9]

Before introducing the formal analysis, we motivate our definitions by considering a game with two principals and two agents, with $S^n = \{1, 2\}$ for both agents. Let $S^n = \{1, 2\}$ for both agents and let 11 be the efficient outcome, and thus the only candidate pure-strategy equilibrium. There are three possible deviation: 12, 21, and 22. Let us combine the following three (IC): $m = 1$ and $s = 22$; $m = 2$ and $s = 12$; $m = 2$ and $s = 21$. Together, they imply

$$G^1_{11} - G^1_{22} + G^2_{11} - G^2_{12} + G^2_{11} - G^2_{21} \geq 0.$$

This is the condition that could not be satisfied in the Opposite Interest Game. We rewrite it in the somewhat cumbersome way:

$$\sum_{s \in S} w^1(s)(G^1_{\bar{s}} - G^1_s) + \sum_{s \in S} w^2(s)(G^2_{\bar{s}} - G^2_s) \geq 0. \tag{11}$$

[8] Groseclose and Snyder [7] present the game in a sequential form. First Principal 1 makes offers. Then, Principal 2 observes the offers made by 1 and makes her offers. They show that a principal may want to buy a *supermajority*, that is, make a positive offer to strictly more than $K + 1$ agents.

[9] For a generic game, there exists a unique efficient outcome and therefore all mixed strategy equilibria are inefficient. For a nongeneric game with more than two efficient outcomes, there may exist an efficient mixed-strategy equilibrium but it would just be a trivial randomization over two or more efficient pure-strategy equilibria.

where

$$w^1(22) = w^2(12) = w^2(21) = 1 \text{ and } w^1(11) = w^1(12) = w^1(21) = w^2(11) = w^2(22) = 0 \quad (12)$$

The idea is that the w's are weights that principals put on possible deviations: $w^m(s)$ is the weight Principal m puts on a deviation from $\hat{s}$ to s. The weights in (12) satisfy

$$w^1(12) + w^1(22) = w^2(12) + w^2(22); \quad (13)$$
$$w^1(21) + w^1(22) = w^2(21) + w^2(22).. \quad (14)$$

Condition (13) says that the sum of weights on deviations which involve the participation of Agent 1 is the same for the two principals. Condition (14) is the same condition for Agent 2.

We make this into a general definition. First, however, an observation. Balanced weights, and later balanced games, are defined with respect to an action profile that is used as a reference point, namely $\hat{s}$. We might indicate this explicitly in the definition, or lighten the definitions and keep this as understood. We choose this second solution, but the reader should be aware of this dependence.

Definition 3 *If agents have only two actions, a vector $(w^m(s))_{m\in M, s\in S}$ is said to be a vector of balanced weights if $w^m(s) \geq 0$ for every n and s, and*

$$\text{for every } m \in M, n \in N \sum_{\{s:s^n\neq\hat{s}^n\}} w^m(s) = \sum_{\{s:s^n\neq\hat{s}^n\}} w^1(s). \quad (15)$$

This definition generalizes (13) and (14), since $\{s : s^n \neq \hat{s}^n\}$ is the set of possible deviations that involve the participation of Agent n. The sum of weights over this set must be constant across principals. The interpretation is that each principal has the same opportunity to influence the action of a particular agent. However, the principal may choose to use this influence over different deviations.[10]

By summing (15) over n, we obtain that if the weight corresponding to each deviation is multiplied by the number of agents that must deviate to realize that deviation, then the sum is constant:

Proposition 3 *If a vector $(w^m(s))_{m\in M, s\in S}$ is a vector of balanced weights, then:*

$$\text{for every } m \in M, \sum_{s\in S} w^m(s)\sharp\{n : s^n \neq \hat{s}^n\} = \sum_{s\in S} w^1(s)\sharp\{n : s^n \neq \hat{s}^n\}.$$

This is in agreement with the previous interpretation. The 'cost' of a deviation is proportional to the number of agents that must be convinced. The total cost must equal the endowment. Hence, deviations that involve a small numbers of agents are cheaper than deviations with many agents. In the Opposite Interest Game, Principal 2 had two cheap deviations and that made it hard for Principal 1 to defend the efficient outcome.

Now, reconsider (eq:inwe). It asks that the sum of benefits from a deviation from 11 to 22, weighted according to a particular vector of balanced weight, be nonnegative. We generalize the condition as follows:

[10] As the simple example of balanced weights (12) shows, the sum over s of the weights needs not be constant: so, in particular, weights *cannot* be interpreted as probabilities.

Definition 4 *A transfer game is balanced if and only if for every vector of balanced weights $(w^m(s))_{m\in M, s\in S}$ we have:*

$$\sum_{m\in M}\sum_{s\in S} w^m(s)(G^m_{\hat{s}} - G^m_s) \geq 0. \tag{16}$$

We are now ready to state our main result:

Theorem 2 *A transfer game where agents have two actions has a pure strategy equilibrium if and only if it is balanced.*

Proof: From proposition 1 we derive that a pure strategy equilibrium exists if and only if the three conditions of that proposition hold. If we denote

$$d^{jn} \equiv t^{jn}_{\hat{s}^n} - t^{jn}_{(\hat{s}^n)'}$$

(AM) and (IC) may be rewritten as

$$\sum_{\{j:j\neq k\}}\ \sum_{\{n:s^n\neq \hat{s}^n\}} d^{jn} \geq G^k_s - G^k_{\hat{s}}, \text{ for every } s\in S, k\in M, \tag{17}$$

and

$$\sum_{j\in M} d^{jn} = 0, \text{ for every } n. \tag{18}$$

The system (17) and (18) is a system of linear inequalities in the $M\times N$ variables d^{jn}. There are $M\times S$ inequalities of the type in (17), each of them indexed by a pair (ms); and N inequalities of the type (18), each indexed by i.

We can find a d that solves (17) and (18) if and only if we can find a t that solves (AM), (IC), and (CM). The "if" part is by definition. The "only if" part can be seen as follows. Suppose we find a d that solves (17) and (18). Let $t^{jn}_{\hat{s}^n} = \max(0, -d^jn)$ and $t^{jn}_{(\hat{s}^n)'} = \max(0, d^jn)$. The resulting t satisfies (AM), (IC), and (CM). Hence, we have shown that there exists a pure-strategy equilibrium with outcome $\hat{s}$ if and only if the system (17) and (18) has a solution.

The following result is useful:[11]

Theorem 3 (Farkas) *Exactly one of the following alternatives is true: (a) There exists a solution x to the linear system of (in)equalities given by $Ax \geq a$ and $Bx = b$; or (b) There exist vectors μ and ν such that: (i) $\mu A + \nu B = 0$; (ii) $\mu \geq 0$; and (iii) $\mu a + \nu b > 0$.*

We now apply Farkas' Lemma to (17) and (18). For $m, j\in M$, $i, n\in N$, $s\in S$, let

$$\begin{aligned}
A_{(ms,jn)} &= \begin{cases} 1 \text{ if } j\neq m, s^n\neq \hat{s}^n, \\ 0 \text{ otherwise;}\end{cases} \\
B_{(i,jn)} &= \begin{cases} 1 \text{ if } j = i, \\ 0 \text{ otherwise.}\end{cases} \\
a_{ms} &= G^m_s - G^m_{\hat{s}} && (19) \\
b_i &= 0 && (20) \\
& && (21)
\end{aligned}$$

[11] See for instance Mangasarian [9].

Then, (17) and (18) rewrite as $Ad \geq a$ and $Bd = b$. By Farkas' Lemma a solution $(d^{jn})_{j\in M, n\in N}$ of that system exists if and only if there is no solution $((w^m(s)_{m\in M, s\in S}, (\nu^i)_{i\in N})$ of the system:

$$\text{for every } j \in M, n \in N, \sum_{m\in M, s\in S} w^m(s) A_{(ms,jn)} + \sum_{i\in N} \nu^i B_{(i,jn)} = 0; \tag{22}$$

$$\text{for every } m \in M s \in S, w^m(s) \geq 0;$$

and

$$\sum_{m\in M} \sum_{s\in S} w^m(s)(G^k_s - G^k_{\hat{s}}) > 0. \tag{23}$$

The system (22) may be rewritten as:

$$\text{for every } j \in M, n \in N, \sum_{\{s: s^n \neq \hat{s}^n\}} w^m(s) = -\nu^n. \tag{24}$$

As this is the only restriction that the variables ν are imposing, (24) is true if and only if w is a vector of balanced weights.

Inequality (23) is the negation that the game is balanced for a particular vector of weights. The lack of solution for the system (22) and (24) is equivalent to the requirement that for all balanced weights the inequality

$$\sum_{m\in M} \sum_{s\in S} w^m(s)(G^m_{\hat{s}} - G^m_s) \geq 0$$

holds, and this is the statement we had to prove. ■

4.4 Remarks

1. Games of common agency are of course a special case of the games we are considering, where $N = \{1\}$. In the case of an agent with two actions, the vector of weights $w^m(s)_{s\in S}$ of the principal m is a scalar, and the condition (15) that they are balanced requires these scalars to the same. So a pure strategy equilibrium giving $\hat{s}$ as equilibrium outcome exists if and only if, for all s,

$$\sum_{m\in M} (G^m_{\hat{s}} - G^m_s) \geq 0,$$

   that is an equilibrium in pure strategies always exists, and it gives the efficient action, which is in accord with Bernheim and Whinston [3].

2. The other extreme case is one principal and many agents, that is $M = \{1\}$. Balancedness means that

$$\sum_{s\in S} w^1(s)(G^1_{\hat{s}} - G^1_s) \geq 0.$$

   for any nonnegative vector $w(s)$. This is equivalent to $G^m_{\hat{s}} - G^m_s \geq 0$ for all s. Hence, again, an efficient equilibrium always exists, which is the result that Segal [12] obtains in absence of agent interdependences.

3. Given a deviation $\tilde{s}$, a possible vector of balanced weights is, for every $m \in M$,

$$w^m(s) = \begin{cases} 1 & \text{if } s = \tilde{s}; \\ 0 & \text{otherwise.} \end{cases} \tag{25}$$

This vector is balanced because each principal is asking exactly the same deviation from all agents. Then, we get that a game is balanced only if, for every $\tilde{s} \in S$,

$$\sum_{m \in M} (G^m_{\hat{s}} - G^m_{\tilde{s}}) \geq 0$$

that is, $\hat{s}$ is the efficient action. This is an indirect way of getting to Proposition 2. Of course, efficiency does not in general imply balancedness. That is because the weights in (25) assume that all principals have the same 'best' deviation and that need not be true.

4. Let $C(s) \equiv \sharp\{s : s^n \neq \hat{s}^n\}$, and NW denote the set of balanced weights, normalized by

$$\sum_{s \in S} w^m(s) C(s) = 1. \tag{26}$$

Since the inequality defining a balanced game is homogeneous, the condition

$$\min_{w \in NW} \sum_{m \in M, s \in S} w^m(s)(G^m_{\hat{s}} - G^m_s) \geq 0 \tag{27}$$

is necessary and sufficient for existence of an equilibrium in pure strategies giving $\hat{s}$ as action profile outcome. Now let

$$C \equiv \{(w^m(s))_{m \in M, s \in S} : w^m(s) \geq 0 \text{ for every } m, s, \text{ and } \sum_{s \in S} w^m(s) C(s) = 1\}.$$

By proposition (3), $NW \subset C$, and therefore a sufficient condition for the existence of equilibria in pure strategies is

$$\min_{w \in C} \sum_{m \in M, s \in S} w^m(s)(G^m_{\hat{s}} - G^m_s) \geq 0$$

But the set C has a product structure: a vector w belongs to C if and only if each m-th component satisfies a set of constraints that only depend on w^m. The minimization problem is equivalent to:

$$\sum_{m \in M} \min_{s \in S} \left(\frac{G^m_{\hat{s}} - G^m_s}{C(s)} \right) \geq 0, \tag{28}$$

so we may state:

Proposition 4 *An equilibrium in pure strategies giving $\hat{s}$ as equilibrium action profile exists if (28) holds.*

5. The following result makes Theorem 2 of immediate use from a computational point of view:

Proposition 5 *If a transfer game is balanced for every vector of balanced weighs $w \in \{0,1\}^{SM}$, then it is balanced for every vector of balanced weights.*

Proof: From Definition 4, a transfer game is balanced if and only if the value of the following linear program is negative:

$$\min_{w,k} \sum_{m \in M} \sum_{s \in S} w^m(s)(G^m_{\hat{s}} - G^m_s)$$

subject to

$$\text{for every } m \in M, n \in N : \sum_{\{s: s^n \neq \hat{s}^n\}} w^m(s) = k^n,$$
$$\text{for every } m \in M, s \in S : w^m(s) \geq 0.$$

This linear program is homogeneous in w and k. Hence, we can assume, without loss of generality, that $\max_{n \in N} k^n = 1$ (if none of the k's is strictly positive, then the proposition holds trivially). Each k^n appears only in one constraint and does not appear in the objective. Therefore, we can assume without loss of generality that, for every $n \in N$, $k^n \in \{0, 1\}$.

Now, for every m and every s, $w^m(s) \in [0, 1]$. Suppose that $w^m(s) \in (0, 1)$. It is feasible to both increase and decrease $w^m(s)$ by varying w^m for $s \neq S$. Hence, as the objective function is linear, we can assume without loss of generality that $w^m(s) \in \{0, 1\}$. ∎

The problem of deciding if a game played through agents has a pure strategy equilibrium is now very simple. First, determine the set of efficient outcomes. Let $\hat{s}$ be an element of this set (it will be the unique element in a generic game). Then, apply Proposition 5 and examine all the possible matrices $w \in \{0,1\}^{S \times M}$. Verify that for each w either w is not balanced or (16) holds. The computation time is linear in the size of the problem (MNS).

5 The General Case

We now leave the simplified environment where agents have only two actions, but we still maintain the restriction that agents' payoffs do not depend on s directly.

If agents have more than two actions, a pure-strategy equilibrium need not be efficient. This is already true if $N = \{1\}$ (common agency). Consider the example (see [3]) where $M = \{1, 2\}, N = \{1\}$ and $\sharp S^n = 4$, with

$$G^1 = (8, 6, 0, 1), G^2 = (0, 6, 7, 1); \tag{29}$$

Here $t^1 = (7, 0, 0, 0)$, $t^2 = (0, 0, 7, 0)$, and $s = 1$ is a pure strategy equilibrium with an inefficient outcome. The main feature of this equilibrium is a failure of the two principals to coordinate on the efficient action. Principal 1 does not make an offer on action 2 because Principal 2 is not making an offer either, and viceversa. There exists another pure-strategy equilibrium in which $t^1 = (3, 1, 0, 0)$, $t^2 = (0, 2, 3, 0)$, and $s = 2$, which selects the efficient action and gives a higher payoff to both principals.

To overcome this multiplicity of equilibria, in common agency Bernheim and Whinston introduce the notion of truthful transfers. A transfer vector is truthful if, for all actions, it is equal to the principal's gross payoff minus a constant (save for the nonnegativity constraint on transfers). Formally,

Definition 5 *If $N = \{1\}$, a transfer vector t^m is truthful relative to $\hat{s}$ if for every $s \in S$*

$$t^m_s = \max(0, t^m_{\hat{s}} + G^m_s - G^m_{\hat{s}})$$

An pure strategy equilibrium giving $\hat{s}$ as equilibrium action is truthful if the strategy of every principal is truthful relative to $\hat{s}$.

In common agency, truthful equilibria play a fundamental role. They always exist, the equilibrium action outcome is efficient ([3, Theorem 2]) and they are coalition proof ([3, Theorem 3]). The intuition is that truthful transfers restrict offers on out-of-equilibrium actions not too be too low with respect to the principals' payoffs and therefore exhausts all gains from coalitional deviations.

But truthful equilibria are very hard to come by in the transfer game. For instance, in the prisoner's dilemma game a vector of truthful strategies should satisfy (disregarding the nonnegativity constraints) $a = y - x, b = y$ for the first principal, and $a = y, b = y - x$ for the second, and this is impossible. Intuitively, the requirement of being truthful imposes too many equations on the strategy.

However, one can relax Definition 5 from equality to inequality. A weaker condition is that for every $s \in S$, $t^m_s \geq t^m_{\hat{s}} + G^m_s - G^m_{\hat{s}}$, or alternatively

$$G^m_s - t^m_s \geq G^m_{\hat{s}} - t^m_{\hat{s}}.$$

This definition maintains the feature that offers on out-of-equilibrium actions cannot be too low and it can be extended to a transfer game with many agents:

Definition 6 *In a transfer game, t^m is weakly truthful relative to $\hat{s}$ if*

(WT) For every $m \in M$ and $s \in S$, $G^m_{\hat{s}} - \sum_{n \in N} t^{mn}_{\hat{s}^n} \geq G^m_s - \sum_{n \in N} t^{mn}_{s^n}$.

A weakly truthful equilibrium is a pure strategy equilibrium giving $\hat{s}$ as equilibrium outcome, and in which the strategy of every principal is weakly truthful relative to $\hat{s}$.

A straightforward consequence of this definition is that – like truthful equilibria of common agency games – weakly truthful equilibria of transfer games are always efficient:

Theorem 4 *The outcome of a weakly truthful equilibrium is efficient.*

Proof: Sum the inequalities (WT) over m. Sum the inequalities AM in Theorem 1 over n. Add the two resulting inequalities. The result is the inequality in (1), which defines efficiency. ■

The pair of action and transfer outcomes of a weakly truthful equilibrium has a simple characterization. The necessary and sufficient conditions for an action profile to be supported by a weakly truthful equilibrium are the same as those for an action profile to be supported by an equilibrium (Theorem 1), except that (IC) is substituted with the stronger requirement that transfers be weakly truthful:

Theorem 5 *A pair $(\hat{t}, \hat{s})$ of transfers and action profiles is the outcome of a weakly truthful equilibrium if and only if they satisfy WT, AM and CM hold.*

Proof: Sum (AM) over n. To the resulting inequality, add (WT). The result is (IC). Hence, (WT) and (AM) imply (IC) and sufficiency is proven. Necessity is obvious because (AM) and (CM) are necessary by Theorem 1 and (WT) is necessary by the definition of weakly truthful equilibrium. ■

To check that the definition of weakly truthful equilibrium is consistent with the analysis of the previous section, consider what happens to weakly truthful equilibria if each agent has only two actions and cares solely about monetary payoff. In this case, it is immediate to check that (AM) and (IC) imply (WT) and, by Theorem 5:

Corollary 2 *if each agent has only two actions and cares solely about monetary payoff, then all equilibria are weakly truthful.*

Weak truthfulness eliminates inefficient equilibria that are due to coordination problems. If each agent has only two actions, coordination problems do not arise because each principal will contribute for either one action or the other. Hence, weak truthfulness has no bite.

We now move to the question of existence. As in the previous section, we pose the question of existence with respect to a particular action profile, that is, we ask whether, given $\hat{s} \in S$, there exists a weakly truthful equilibrium that produces outcome $\hat{s}$. Let us redefine balanced weights as follwos:

Definition 7 *The vector w with dimension MS is a vector of balanced weights if*

$$\text{for every } m \in M, n \in N, a^n \in S^n/\hat{s}^n : \sum_{\{s:s^n=a^n\}} w^m(s) = \sum_{\{s:s^n=a^n\}} w^1(s).$$

If agents have only two actions, there is only one possible deviation for each agent. With more than two actions, the condition that principals put the same sum of weight must hold for every agents *and* for every deviation that the agent has.

Let the definition of balanced game be exactly the same as in Definition 4. We are ready to state the main result of this section:

Theorem 6 *A transfer game has a weakly truthful equilibrium if and only if it is balanced.*

Proof: At this point, to prove existence we need to show that there exists a vector of transfers, satisfying the (WT), (AM), and (CM). However, we can simplify our task by rewriting $t^{mn}_{s^n} - t^{mn}_{\hat{s}^n}$ as $d^{mn}_{s^n}$ and showing:

Proposition 6 *There exists a weakly truthful equilibrium with outcome $\hat{s}$ if and only if there exists $d \in R^{MH}$ that satisfies*

(WTd) For all $s \in S$ and all $m \in M$,

$$\sum_{n:s^n \neq \hat{s}^n} d^{mn}_{s^n} \geq G^m_s - G^m_{\hat{s}};$$

(AMd) For all $n \in N$ and all $s^n \in S^n$,

$$\sum_{m \in M} d^{mn}_{s^n} \leq 0.$$

Proof of Proposition 6 Let $d^{mn}_{s^n} \equiv t^{mn}_{s^n} - t^{mn}_{\hat{s}^n}$. Then, (WTd) is (WT) and (AMd) is (AM). Hence, the "only if" part is immediate.

To prove sufficiency, suppose that a vector d has been found that satisfies (WTd) and (AMd). Clearly, there exists a vector t that satisfies (WT) and (AM). For every m and n, take $\{t^m n_{s^n}\}_{s^n \in S^n}$ and decrease each element $t^{mn}_{s^n}$ of the same amount (unless $t^{mn}_{s^n} = 0$) until one of the following is true:

either $\quad t^{mn}_{\hat{s}^n} = 0$;

or $\quad$ there exists $\bar{s}^n \neq \hat{s}^n$ such that $\sum_{j \in M} (t^{jn}_{\hat{s}^n} - t^{jn}_{\bar{s}^n}) = 0$ and $t^{mn}_{\bar{s}^n} = 0$.

Repeat this operation for all $m \in M$. This does not interfere with (WT) because it either decreases both sides of the inequality of the same amount or it only decreases the left side. Neither does it interfere with any of the (AM) because, for each n, it halts as soon as soon as one of the inequalities becomes an equality. The new $\hat{t}$, found in this manner, satisfies (CM) by construction.

With Proposition 6, we can focus on necessary and sufficent conditions for the existence of a vector d that solves (WTd) and (AMd). We use the following:

Theorem 7 (Gale's Theorem of the Alternative) [12] *Given a matrix A and a vector a, either (i) there exists an x such that $Ax \geq a$; or (ii) there exists a y such that $yA = 0$, $yc < 0$, and $y \geq 0$.*

We rewrite (WTd) and (AMd) in a way that fits (i) of Theorem 7. Let

$$\begin{aligned} B_{(ms,jna)} &= \begin{cases} -1 \text{ if } j = m, s^n = a, s^n \neq \hat{s}^n \\ 0 \text{ otherwise;} \end{cases} \\ C_{(ns^n,jia)} &= \begin{cases} 1 \text{ if } n = i, s^n = a, \\ 0 \text{ otherwise.} \end{cases} \\ b_{ms} &= G^m_{\hat{s}} - G^m_s && (30) \\ c_i &= 0 && (31) \\ & && (32) \end{aligned}$$

Then B has dimensions (MS, MH), C (H, MH), a $(MS, 1)$, and b $(H, 1)$. If we let

$$A = \begin{bmatrix} B \\ C \end{bmatrix},$$

and

$$a = \begin{bmatrix} b \\ c \end{bmatrix},$$

[12] See Mangasarian [9, p. 33].

we get (i) of Theorem 7.

By Theorem 7, (i) is true if and only if there is no y such that (ii) is true. Let $y = [w, z]$, where w has dimensions $(1, M \times S)$ and z $(1, H)$. Then (ii) says that $wB+zC = 0$, $wb+zc < 0$, $w, z \geq 0$.

Let $1_{(\cdot)}$ be the indicator function. The system $wB + zC = 0$ can be rewritten as: for every $m \in M, n \in N$ $a^n \in S^n$:

$$-\sum_{j \in M} \sum_{s \in S} w^m(s) 1_{(m=j, s^n \neq \hat{s}^n, s^n = a^n)} + \sum_{i \in N} \sum_{a^n \in S^n} z^n(s^n) 1_{(i=n, s^n = a^n)} = 0,$$

which we can write

$$\text{for every } m \in M, n \in N, a^n \in S^n/\hat{s}^n : \sum_{\{s: s^n = a^n\}} w^m(s) = z^n(a^n).$$

As the right side does not depend on m, the left side must be independent of m as well. Note also that z is not subject to other restrictions. Hence, we can write

$$\text{for every } m \in M, n \in N\, a^n \in S^n/\hat{s}^n : \sum_{\{s: s^n = a^n\}} w^m(s) = \sum_{\{s: s^n = a^n\}} w^1(s).$$

and $wB + zC = 0$ is equivalent to the requirement that weights be balanced.

The system $wb + zc < 0$ reduces to $wb < 0$ because $c = 0$, and can be easily transformed into

$$\sum_{m \in M} \sum_{s \in S} w_m(s)(G^m_{\hat{s}} - G^m_s) < 0. \tag{33}$$

Statement (ii) of Theorem 7 is false (and a weakly truthful equilibrium exists), if and only if, for all vectors of balanced weights w, inequality (33) is never satisfied. That is,

$$\sum_{m \in M} \sum_{s \in S} w_m(s)(G^m_{\hat{s}} - G^m_s) \geq 0,$$

which is Definition 4. ■

6 Agents with Direct Preferences

So far, agents have only cared about money. In this section, agents also care for the action that is chosen. Agent n's utility function is

$$U_i(t, s) = \sum_{m \in M} t^{mn}_{s^n} + F^n_s.$$

No restriction is placed on F.[13] Thus, the direct utility that the agent receives from s may depend both on the component under the control of the agent (s^n) and the component under the control of other agents (s^{-n}).

We assume that transfers are secret. An agent observes the transfers offered to him but not the transfers offered to the other agents. Agent n thus observes $t^n \equiv \{t^{mn}_{s^n}\}_{(m \in M, s^n \in S^n)}$

[13]The separability assumption is probably not essential. As Dixit, Grossman, and Helpman [6] have shown, the crucial results of common agency are still valid if the agent has a nonseparable utility function.

and forms beliefs $\tau(t^n)$ about the transfers made to the other agents. We simplify the problem by restricting beliefs to be passive:[14]

Definition 8 *Agents hold passive beliefs if $\tau(t^n)$ is independent of t^n. A passive belief equilibrium is a pure-strategy perfect Bayesian equilibrium of the transfer game in which agents hold passive beliefs.*

If agent n holds a passive beliefs and he observes a deviation from one of the principals, he assumes that the principal has deviated only with him but is still offering the same transfers to the other agents. A passive belief equilibrium is then simply a pair $(\hat{t}, \hat{s})$.

Both the assumptions of private offers and passive beliefs can be seen as arbitrary. Other assumptions are possible, and maybe more appropriate in certain circumstances. We do not aspire to provide a comprehensive treatment of multi-principal multi-agent games with externalities among agents. Our goal in this section is simply to show that the logic of the results we have obtained in the previous sections can be fruitfully extended to the case in which agents have direct preferences.

Let a *weakly truthful passive belief equilibrium* be a passive belief equilibrium in which (WT) is satisfied (as (WT) does not depend on the agents' preferences, Definition 5 applies this section as well). Let $(s^n, \hat{s}^{-n}) \equiv (\hat{s}^1, \ldots, s^n, \ldots, \hat{s}^N)$ be the outcome when all agents play $\hat{s}^j$ but agent n deviates to s^n. We can now show that the equilibrium characterization given in the previous sections is still valid, albeit with small modifications:

Theorem 8 *A pair $(\hat{t}, \hat{s})$ of transfers and action profiles is the outcome of a weakly truthful passive belief equilibrium if and only if the following conditions are satisfied:*

(AM') for every $n \in N$, $s^n \in S^N$,

$$\sum_{m \in M} \hat{t}^{mn}_{\hat{s}^n} + F^n_{\hat{s}} \geq \sum_{m \in M} \hat{t}^{mn}_{s^n} + F^n_{(s^n, \hat{s}^{-n})};$$

(IC') for every $m \in M$, $s \in S$,

$$G^m_{\hat{s}} + \sum_{n \in N} \sum_{j \neq m} \hat{t}^{jn}_{\hat{s}^n} + \sum_{n \in N} F^n_{\hat{s}} \geq G^m_s + \sum_{n \in N} \sum_{j \neq m} \hat{t}^{jn}_{\hat{s}^n} + \sum_{n \in N} F^n_{(s^n, \hat{s}^{-n})};$$

(CM') for every $m \in M$, $n \in N$,

$$\sum_{j \in M} \hat{t}^{jn}_{\hat{s}^n} + F^n_{\hat{s}} = \max_{a \in S^n} [\sum_{j \neq m} \hat{t}^{jn}_a + F^n_{(a, \hat{s}^{-n})}].$$

Proof: The proof is exactly the same as the proof of Theorem 1, except that $T^{mn}_{s^n} \equiv \sum_{j \neq m} \hat{t}^{jm}_{s^n}$ is replaced with $T^{mn}_{s^n} \equiv \sum_{j \neq m} \hat{t}^{jm}_{s^n} + F^n_{(s^n, \hat{s}^{-n})}$. ■

As a limit case, consider what happens to the conditions of Theorem 8 when there are no principals ($M = \emptyset$). The conditions (AM'), (IC'), and (CM') reduce to: for every $n \in N$ and

[14]The restriction to passive beliefs is common in the literature on principal-agent models with one principal and many agents. See McAfee and Schwartz [10] for a discussion and further references.

$s \in S$, $F_{\hat{s}}^n \geq F_{(s^n,\hat{s}^{-n})}^n$. This is the necessary and sufficient condition for $\hat{s}$ to be the outcome of a Nash-equilibrium in the game played among agents.[15]

As in the previous sections, we study conditions for existence. First, we need to redefine balancedness. The new definition is:

Definition 9 *In a transfer game with direct agent preferences, the vectors w and z with respective dimensions MS and H are said to be vectors of balanced weights if all their elements are nonnegative, and*

$$\text{for every } m \in M, n \in N, a^n \in S^n/\hat{s}^n; \sum_{\{s:s^n=a^n\}} w^m(s) = z^n(a^n).$$

A transfer game with direct agent preferences is balanced if and only if for every pair of vectors of balanced weights w and z we have:

$$\sum_{m\in M}\sum_{s\in S} w^m(s)(G_{\hat{s}}^m - G_s^m) + \sum_{n\in N}\sum_{s^n\in S^n} z^n(s^n)(F_{\hat{s}}^n - F_{(s^n,\hat{s}^{-n})}^n) \geq 0$$

The definition of balancedness now includes weights on agents as well as principals. This is because, if we are considering a deviation from $\hat{s}$ to s, we have to take into account not only the benefit of principals but also that of agents. A game is balanced (with respect to a given action $\hat{s}$) if, for any vector of balanced weights, the sum of the direct change in payoffs for principals and agents of any possible deviation is negative. If $F_s^n = 0$ for all agents and all actions, we recover the definition of balancedness used in the previous section.

The main result of this section is:

Theorem 9 *A transfer game with agent preferences has a weakly truthful passive belief equilibrium if and only if it is balanced.*

Proof: The analogous of Proposition 6 can be proven for the case with externality (the proof of this proposition follows familiar lines and is omitted):

Proposition 7 *There exists a weakly truthful equilibrium with outcome $\hat{s}$ if and only if there exists $d \in R^{MH}$ that satisfies*

(WTd') For all $s \in S$ and all $m \in M$,

$$\sum_{n:s^n\neq\hat{s}^n} d_{s^n}^{mn} \geq G_s^m - G_{\hat{s}}^m;$$

(AMd') For all $n \in N$ and all $s^n \in S^n$,

$$\sum_{m\in M} d_{s^n}^{mn} \leq F_{\hat{s}}^n - F_{(s^n,\hat{s}^{-n})}^n.$$

[15] It is also easy to see that, if we take a transfer game and let all the G's tend to zero uniformly (while keeping the F's constant), we obtain that the limit of (AM'), (IC'), and (CM') is the Nash condition. This suggests a degree of continuity between the concept of weakly truthful passive belief equilibrium and the concept of Nash equilibrium. An equilibrium concept that included some form of coordination among agents may not enjoy this property.

We rewrite (WTd') and (AMd') in a way that fits (i) of Theorem 7. Let B, C, and b be like in the proof of Theorem 6 but let, for all n and all $s^n \in S^n$,

$$c_{ns^n} = F^n_{\hat{s}^n} - F^n_{(a,\hat{s}^{-n})}.$$

If we let

$$A = \begin{bmatrix} B \\ C \end{bmatrix},$$

and

$$a = \begin{bmatrix} b \\ c \end{bmatrix},$$

we get (i) of Theorem 7.

By Theorem 7, (i) is true if and only if there is no y such that (ii) is true. Let $y = [w, z]$, where w has dimens ons $(1, M \times S)$ and z $(1, H)$. Then (ii) says that $wB + zC = 0$, $wb + zc < 0$, $w, z \geq 0$.

As B and C are the same as in the proof of Theorem 6, the system $wB + zC = 0$ is equivalent to the requirement that

$$\text{for every } m \in M, n \in N, a^n \in S^n/\hat{s}^n : \sum_{\{s: s^n = a^n\}} w^m(s) = z^n(a^n).$$

which is the definition of balanced weights when agents have direct preferences.

The system $wb + zc < 0$ can be transformed into

$$\sum_{m \in M} \sum_{s \in S} w_m(s)(G^m_{\hat{s}} - G^m_s) + \sum_{n \in N} \sum_{s^n \in S^n} z^n(s^n)(F^n_{\hat{s}} - F^n_{(a,\hat{s}^{-n})}) < 0,$$

which completes the proof. ■

When agents have no direct preferences, it is not true anymore that balancedness implies efficiency. There can exist a weakly truthful equilibrium which is not efficient. This, in turn, implies that weakly truthful equilibria are not generically unique anymore. The following example illustrates both these facts:

Example Consider a game with two agents, two principals, and two actions per agent. Each agent can say "red" or "blue". The agents' preferences are: $(F^1_{r,r} = 1, F^2_{r,r} = 1)$; $(F^1_{r,b} = 0, F^2_{r,b} = 0)$; $(F^1_{b,r} = 0, F^2_{b,r} = 0)$; $(F^1_{b,b} = 2, F^2_{b,b} = 2)$. The principals' preferences are: $(G^1_{r,r} = x, G^2_{r,r} = 0)$; $(G^1_{r,b} = 0, G^2_{r,b} = 0)$; $(G^1_{b,r} = 0, G^2_{b,r} = 0)$; $(G^1_{b,b} = 0, G^2_{b,b} = x)$, where $x \geq 0$. Hence, the efficient outcome is (r, r). It is easy to see that, if $x \leq 2$, both (b, b) and (r, r) satisfy balancedness and, hence, there are two weakly truthful equilibria outcomes. Agents are playing a pure coordination game with two equilibria: one efficient and one inefficient. When $x > 2$, the equilibrium with (b, b) disappears. Only if principals have enough interest in the game, the inefficient equilibrium disappears.

However, there exists a simple (but strong) sufficient condition to restore efficiency and uniqueness:

Corollary 3 *If there exists an efficient action s^* such that, for all $n \in N$ and all $s \in S$, $F^n_{(s^{*n}, s^{-n})}$ does not depend on s^{-n}, then all weakly truthful passive belief equilibria are efficient.*

Proof: Let $\hat{s}$ be the outcome of a weakly truthful passive belief equilibrium. Consider a vector of balanced weights in which $w^m(s) = 1$ if $s = s^*$ and $z^n(s^n) = 1$ if $s^n = s^{*n}$, with all the other weights equal to zero. For this set of weights, balancedness requires

$$\sum_{m \in M} \sum_{s \in S} (G^m_{\hat{s}} - G^m_{s^*}) + \sum_{n \in N} (F^n_{\hat{s}} - F^n_{(s^*, \hat{s}^{-n})}) \geq 0$$

which, because of the assumption of the corollary, rewrites as

$$\sum_{m \in M} \sum_{s \in S} (G^m_{\hat{s}} - G^m_{s^*}) + \sum_{n \in N} (F^n_{\hat{s}} - F^n_{s^*}) \geq 0$$

This is true only if $\hat{s}$ is efficient. ■

The last corollary is the generalization of Segal [12, Proposition 3] to a multiple-principal environment. If there is only one principal, it is immediate to see that all equilibria are weakly truthful (or do not differ from a weakly truthful equilibrium in a payoff-relevant way). Then, Corollary 3 reduces to: If there exist an efficient action s^* such that, for all $n \in N$ and all $s \in S$, $F^n_{(s^{*n}, s^{-n})}$ does not depend on s^{-n}, then all equilibria are efficient.

7 Mixed-Strategy Equilibria

We have seen several examples, by now, showing that existence of equilibria is not ensured with pure strategies only. We provide here a general existence result. To do this, we define the problem as a game with endogenous sharing rules. (see Simon and Zame, [13]). We recall that an M-players game with an endogenous sharing rule is a tuple $\Gamma = (T^1, \ldots, T^M, Q)$ consisting of a strategy space T^m for each player, and a payoff correspondence $Q : T^1 \times \ldots T^M \rightarrow R^M$. A sharing rule is a Borel measurable selection from the correspondence Q; *i.e.* is a Borel measurable function $q : T \rightarrow R^M$ such that $q(t) \in Q(t)$ for each $t \in T$. A solution for Γ is a sharing rule and a mixed strategy profile such that, given the sharing rule, each player's action is a best response to the mixed strategy of the other players.

Simon and Zame [13] provide a general existence result for a large class of games with endogenous sharing rules that satisfy the following conditions:

1. there is a dense subset T^* of the product of the strategy spaces T, and a bounded continuous function $\phi : S^* \rightarrow R^M$;

2. C_ϕ is the correspondence whose graph is the closure of the graph of ϕ, and $Q(t)$ is the convex hull of $C_\phi(t)$ for each t.

The following existence theorem is an immediate consequence of the main theorem of Simon and Zame [13]:

Theorem 10 *The transfers game has a solution in mixed strategies.*

Proof: In our game, the players are the principals; the strategy space T^m of each principal is the set $[0, K]$. The set T^* is the set $t \in T : \sum_{m \in M} t^{mn}_{s^n} \neq \sum_{m \in M} t^{mn}_{s^n_0}$ for every pair of actions s^n, s^n_0 and every n; that is, the set of transfers of each principal such that the best choice of action of each player is uniquely defined. This is clearly a dense subset of the space of transfers. Let the function ϕ be defined for every $t \in T^*$ as $\phi(t) \equiv G^m_{s^*}$ where for each agent s^{*n} is the action that maximizes the payoff of the agent. In addition, let the agents use the set of correlated strategies on the actions that give equal payoff, at any vector of transfers of the principals where this occurs. This is the convex completion of the function ϕ. It is now immediate to check that all the conditions of the general existence theorem of Simon and Zame are satisfied, hence an equilibrium exists. ∎

A transfer game is a two-stage game. Given the principals' transfers, the second stage is a finite game played among the agents. Let $S(t)$ denote the set of second stage equilibria given t. It is easy to see that S is nonempty. Moreover, with passive beliefs, for almost all t, S is continuous in t (because each agent strictly prefers one action over the others). We can view the principal stage as a game with M players in which $S(t)$ is given. This game satisfies Simon and Zame's conditions for a game with endogenous sharing rules and therefore is guaranteed to have an equilibrium. The equilibrium will include a sharing rule that dictates what each agent should do in case she is indifferenet between two actions.

To illustrate mixed-strategy equilibria, reconsider the example of Opposite Interest Game of Section 4. For that game, we have shown that there does not exist a pure-strategy equilibrium. Theorem 10 guarantees the existence of a mixed strategy equilibrium. It turns out that there exists a mixed-strategy equilibrium as follows: The first principal makes a transfer

$$(0, s, 0, 0) \text{ with probability } (1/2)F(s) = \frac{s}{3-s}$$

and a tranfer

$$(0, 0, 0, s) \text{ with probability } (1/2)F(s) = \frac{s}{3-s},$$

in both cases with $s \in [0, 3/2]$. The second principal makes a tranfer

$$(t, 0, t, 0) \text{ with probability } G(t) = \frac{3/2}{2-t}$$

again with $t \in [0, 3/2]$.

8 Sequential Version

In the trasfer game that has been considered so far, all principals play simultaneously. One may suspect that it is this simultaneity that drives the inefficiency results. We devote this section to variations of the original game in which principals make their offers sequentially rather than simultaneously. The goal of the section is to show that our inefficiency results are robust.

The transfer game that we have used so far is simultaneous in two ways: (1) All principals make their offers to one specific agent simultaneously; (2) A specific principal makes his offers to all agents simultaneously. In the next two sections we modify (1) and (2) one at a time. In Subsection 8.1 we eliminate (1) and assume instead that principals take turns making their offers. Principal m goes first, then Principal $m-1$, all the way down to principal 1. In the end, agents make their choices. We refer to this timing as *principal sequentiality.* Instead, in Subsection 8.2 we eliminate (2) and we assume that all principals make offers to one agent, who chooses his action, then all principals make offers to another agent, who makes his choice, and so on for all agents. We label this second type of sequential timing ***agent sequentiality.***

Of the three examples discussed in the introduction: lobbying has been modeled both in a simultaneous fashion (e.g. Dixit, Grossman, and Helpman [6]) and in a principal-sequential manner (Groseclose and Snyder [7]); supply contracts have been studied in the simultaneous framework (Bernheim and Whinston [4]) and in a principal-sequential framework (e.g. Aghion and Bolton [1]) with the idea that one manufacturer is the incumbent and enjoys a first-mover advantage; multiple auctions are naturally studied in an agent-sequential model, although we are aware of no such work.

8.1 Principal Sequentiality

Action sets of the agents and of the principals are defined as in the common agency game we have considered so far. But in the sequential common agency game principals move sequentially.

The M-th principal moves first and principal 1 moves last. Each principal announces a vector $t^m \in R^S$ of transfers to each agent; this announcement is commonly observed. Then the next principal does the same. Finally, agents move simultaneously choosing the action.

The equilibrium set can be characterized using the basic idea of principal agent problems. For every $m \in \{1, \ldots, M\}$ and any vector of transfers $(t^k, \ldots, t^M)$, we denote the subgame beginning after that vector of transfers has been announced by $\Gamma(t^k, \ldots, t^M)$. For any such vector of transfers, and for every subgame-perfect equilibrium (SPE) of the game induced by this vector there is a set of actions chosen as equilibrium outcome of that subgame. We denote the set of all such actions as $\eta(t^k, \ldots, t^M)$.

Take the principal m who is moving in the subgame $\Gamma(t^{m+1}, \ldots, t^M)$. We may think that in solving the backwards induction problem this principal is choosing his transfer t^m *and* the action profile of the agents, provided the choice of this pairs satisfies the incentive constraint that the chosen action is an equilibrium in the subgame beginning at $(t^m, \ldots, t^M)$.

So m is solving the problem:

$$\max_{s \in S}(G_s^m - \sum_{n \in N} t_{s^n}^{mn}) \tag{34}$$

subject to the constraint that :

$$s \in \eta(t^m, \ldots, t^M).$$

To study the equilibria of the game, consider first the principal who is moving last (that is, the principal 1). He is taking as given the vector $(t^2, \ldots, t^M)$ of transfers of the previous principals, and is solving:

$$\max_{s^1 \in S, t^1 \in R^C}(G_{s^1}^1 - \sum_{n \in N} t_{s_n^1}^{1n}) \tag{35}$$

subject to:

$$s^1 \in \eta(t^1, \ldots, t^M). \tag{36}$$

If we denote by $C(t^2, \ldots, t^M, s^1)$ the minimum cost for principal 1 to implement s^1, that is the value of the problem:

$$\min_{t^1 \in R^C} \sum_{n \in N} t_{s_n^1}^{1n}, \text{ subject to } s^1 \in \eta(t^1, \ldots, t^M), \tag{37}$$

the problem of principal 1 is equivalent to

$$\max_{s^1} G_{s^1}^1 - C(t^2, \ldots, t^M, s^1). \tag{38}$$

But $s^1 \in \eta(t^1, \ldots, t^M)$ if and only if s^1_n maximizes the payoff to agent n for every n, that is if and only if:

$$G^{0n}_{s^1_n} + \sum_{j=1}^{M} t^{jn}_{s^1_n} \geq G^{0n}_{s'_n} + \sum_{j=1}^{M} t^{jn}_{s'_n} \tag{39}$$

for every $s'_n \in S^n$ and every n. Hence it is easily seen that:

$$C(t^2, \ldots, t^M, s^1) = \sum_n [\max_{s'_n} \left(G^{0n}_{s'_n} + \sum_{j=2}^{M} t^{jn}_{s'_n} \right) - \left(G^{0n}_{s^1_n} + \sum_{j=2}^{M} t^{jn}_{s^1_n} \right)]. \tag{40}$$

The term $\sum_n \max_{s'_n} \left(G^{0n}_{s'_n} + \sum_{j=2}^{M} t^{jn}_{s'_n} \right)$ is a constant in s^1, and therefore the set of actions chosen at a SPE by the principal 1 in the subgame $\Gamma(t^2, \ldots, t^M)$ is the set of solutions of

$$\max_{s^1} (G^1_{s^1} + \sum_{n \in N} (G^{0n}_{s^1_n} + \sum_{j=2}^{M} t^{jn}_{s^1_n})). \tag{41}$$

In the case of the sequential game with a single agent, the reasoning above extends to all the principals. In fact one can prove:

Proposition 8 *For any m and any $(t^{m+1}, \ldots, t^M)$, the action s^m is a solution of the problem:*

$$\max_{s^m} (\sum_{j=0}^{m} G^j_{s^m} + \sum_{j=m+1}^{M} t^j_{s^k}) \tag{42}$$

if and only if $s^m \in \eta(t^{m+1}, \ldots, t^M)$ for a SPE of the game $\Gamma(t^{m+1}, \ldots, t^M)$.

Here t^j denotes the vector of transfers (one transfer for each action) to the single agent. A corollary of this proposition is obtained considering the case $m = M$. In this case the proposition implies that all the SPE equilibrium outcomes of the sequential game with one agent are efficient:

Theorem 11 *If there is only one agent, in any SPE the agent chooses the efficient action.*

Details of the proof are given in Prat and Rustichini [11], who consider the single agent game extensively. The proposition 8 does *not* generalize to the case of many agents. To see why, consider the problem of the principal 2, the second to last to move. For a given vector $(t^3, \ldots, t^M)$ the problem of minimum cost to implement an action profile $\hat{s}$ is $\min_{t^2} \sum_{n \in N} t^{2n}_{\hat{s}}$ subject to the constraint that $\hat{s}$ solves (41), that is subject to:

$$G^1_{\hat{s}} + \sum_{n \in N} (G^{0n}_{s^1_n} + \sum_{j=2}^{M} t^{jn}_{\hat{s}_n}) \geq G^1_{s'} + \sum_{n \in N} (G^{0n}_{s^1_n} + \sum_{j=2}^{M} t^{jn}_{s'_n}) \tag{43}$$

for every s'. This is a form different from the one in 39 for the last principal. So the minimum cost has a form different from 40. The formula for the minimum cost in a special case is given below.

To analyze the above problem, let:

$$F_s \equiv G_s^1 + \sum_{n\in N}(G^{0n}_{s^1_n} + \sum_{j=3}^{M} t^{jn}_{s_n})$$

For the given vector $\hat{s}$ we denote, for each $s \in S$, $D(s) \equiv \{n \in N : s^n \neq \hat{s}^n\}$, and for any matrix $(F_s)_{s\in S}$

$$F_I \equiv \max_{\{s:D(s)\subseteq I\}} F_s.$$

Lemma 2 *The value of the cost minimization problem:*

$$\min_{t\in R^C} \sum_{n\in N} t^n_{\hat{s}^n} \tag{44}$$

subject to:

$$F_{\hat{s}} + \sum_{n\in N} t^n_{\hat{s}^n} \geq F_s + \sum_{n\in N} t^n_{s^n} \textit{ for all } s \in S$$

is the same as the value of the problem

$$\min_{x\in R^N} x^N \tag{45}$$

subject to:

$$x^I \geq F_I - F_{\hat{s}} \textit{ for all } I \subseteq N, \tag{46}$$

where $x^I \equiv \sum_{n\in I} x^n$.

Note that the probl m 45 defines the *least core* of the game where thevalue of the coalition I is $F_I - F_{\hat{s}}$.

Proof. Note first that if $\hat{t}$ is a solution of the problem (44), then so is the vector defined by

$$\begin{aligned} t^n_{\hat{s}^n} &= \hat{t}_{\hat{s}^n}, \\ &= 0 \text{ if } s^n \neq \hat{t}^n. \end{aligned} \tag{47}$$

So we assume without loss of generality that the solution $\hat{t}$ of the problem (44) satisfies the condition (47). We call now x^n the non-zero coordinate of the vector $\hat{t}$, that is $\hat{t}_{\hat{s}^n}$. The problem (44) is therefore equivalent to

$$\min_{x} \sum_{n\in N} x^n,$$

subject to

$$F_{\hat{s}} + \sum_{n\in N} x^n \geq F_s + \sum_{\{n:s^n=\hat{s}^n\}} x^n, \text{ for all } s \in S. \tag{48}$$

Now observe that

$$\begin{aligned} \max_s(F_s + \sum_{\{n:s^n=\hat{s}^n\}} x^n) &= \max_{\{I\subseteq N\}}(\max_{\{s:D(s)\subseteq I\}} F_s + x_I) \\ &= \max_{\{I\subseteq N\}}(F_I + x^I), \end{aligned} \tag{49}$$

since x is non-negative. So (48) is equivalent to

$$x^N + F_{\hat{s}} \geq F_I + x^I \text{ for all } I \subseteq N$$

hence our claim. ∎

In the case of the game with two principals from the previous discussion we have:

Proposition 9 *In the game with two principals and two agents, the action profile $\hat{s}$ is an equilibrium outcome if and only if it is the solution of*

$$\max_{s^2} G^2_{s^2} - \sum_n t^{2n}_{s^2_n} \tag{50}$$

subject to:

$$\sum_n t^{2n}_{s^2_n} \geq (G^1_{s'} + \sum_n G^{0n}_{s'_n}) - (G^1_{s^2} + \sum_n G^{0n}_{s^2_n}) + \sum_n t^{2n}_{s'_n} \tag{51}$$

for every s'.

Consider now the problem of lemma (2) in the case of two agents. It is easy to see that (writing $a_I \equiv F_I - F_{\hat{s}}$ the problem

$$\min x^{12} \text{ sub. to } x^1 \geq a_1, x^2 \geq a_2, x^{12} \geq a_{12},$$

has value:

$$\max\{a_1 + a_2, a_{12}\}.$$

This gives an explicit solution to the cost minimzation problem of principal 2. Let for any matrix payoff G^i

$$RG_{(s^1,s^2)} \equiv \max_{\{s' \in S^2\}} G^i_{(s^1,s')};\; CG^i_{(s^1,s^2)} \equiv \max_{\{s' \in S^1\}} G^i_{(s',s^2)};$$

and

$$MG^i_{(s^1,s^2)} \equiv \max_{\{s' \in S^1, s'' \in S^2\}} G^i_{(s',s'')};$$

the matrices obtained taking the maximum along rows and columns and the overall maximum, respectively, and the matrix

$$B^i_s \equiv \max\{MG^i_s, RG^i_s + CG^i_s - G^i_s.\}$$

The transfer of minimum cost for principal 2 among those that make principal 1 choose the action profile s is easily found to be, from lemma (2),

$$\max\{MG^1_s - G^1_s, RG^1_s + CG^1_s - 2G^1_s.\}$$

An easy computation now shows that:

Corollary 4 *The action profile $\hat{s}$ is an equilibrium outcome of the sequential game if and only if it solves:*

$$\max_s(\sum_n G^{0n}_{s_n} + G^1_s + G^2_s - B^1_s) \tag{52}$$

In particular a sufficient condition for the equilibrium outcome to be efficient is that the strategic bias matrix B^1_s is constant.

Example If the payoff matrices are:

$$\begin{array}{ccc} & L & R \\ T & 3,0 & 0,2 \\ B & 0,2 & 0,2 \end{array} \tag{53}$$

then the matrix B_s^1 for G^1 is

$$\begin{array}{ccc} & L & R \\ T & 3 & 3 \\ B & 3 & 3 \end{array}$$

so the equilibrium is efficient if principal 1 is the last to move. The equilibrium is inefficient if principal 2 is the last to move; in this case the B^2 matrix is

$$\begin{array}{ccc} & L & R \\ T & 4 & 2 \\ B & 2 & 2 \end{array}$$

The reason for the inefficiency is clear: when the principal 1 moves first, he will only make transfers, if any, on the first action of both agents. But the sum of these transfers can at most be 3, at equilibrium, since this principal can always insure a non-negative payoff. In particular, at least one of the two transfers must be less than 2. But then the principal 2 can get a gross payoff of 2, and a positive net payoff, rather than zero by beating such transfer.

One may conjecture that for every transfer game played in a principal-sequential fashion there exists an ordering of principals such that the game has an efficient subgame-perfect equilibrium. However, this conjecture is incorrect, as the following example shows. Each agent has three actions and the payoff function is:

$$\begin{array}{cccc} & L & C & R \\ T & 3,3 & 0,5 & 5,0 \\ M & 0,5 & 0,0 & 0,0 \\ L & 5,0 & 0,0 & 0,0 \end{array}$$

For both possible ordering of principals, if the efficient outcome (T, L) were supported by a SPE, the first moving principal would have to pay at least 2+2 to guard against deviations by the second mover, but this is clearly not optimal because she can always get at least zero.

8.2 Agent Sequentiality

In the case of agent sequentiality, the timing is as follows. All principals make offers to Agent 1 and Agent 1 chooses an action s_1. Then, all principals learn what action Agent 2 has chosen and make offers to Agent 2, who in turn chooses an action s_2. The game continues in this fashion until the last agent, n. When all agents have chosen their actions, Principal $m \in M$ receives net payoff $G_s^m - \sum_{n \in N} t_{s_m}^{mn}$. The transfer game with agent sequentiality can be seen as a sequence of M common agency games. Each common agency game has the same principals but different agents.

Example Let us consider the already familiar Opposite Interest Game with two principals, two agents, and two actions per agent, that has the following payoff matrix:

$$\begin{array}{ccc} & L & R \\ T & 0,3 & 2,0 \\ B & 2,0 & 2,0 \end{array} \tag{54}$$

In the simultaneous case, we have seen that the Opposite Interest Game does not a have a pure-strategy equilibrium. Hence, the outcome is inefficient. We now look at the agent-sequential version to see if the inefficiency persists. Given the symmetry of the payoff matrix with respect to the two agents, the order of the agents is inconsequential. For concreteness, assume that principals first make offers to Agent 1, then to Agent 2.

The outcome of a subgame perfect equilibrium of the agent-sequential version of the Opposite Interest Game cannot be (T, L). To see this, examine the two possible subgames of the second stage. If Agent 1 has chosen B, then in the second stage both principals are indifferent as to what Agent 2 chooses. If Agent 2 has chosen T, then the second stage is an auction in which principal 1 has valuation 2 and Principal 2 has valuation 3. Principal 2 wins and must pay at least 2. Thus, we can substitute the second stage payoffs in the first stage. If Agent 1 chooses T, the payoffs are $(0, 1)$. If he chooses B, they are $(2, 0)$. Thus, the first stage is an auction in which Principal 1 has valuation 2 and Principal 2 has valuation 1. Principal 1 must win, which shows that the outcome of a SPE cannot be (T, L).

This example shows that the inefficiency that is present in simultaneous transfer games not only persists in the agent-sequential case but it can also become more severe. In the Opposite Ineterest Game, in the simultaneous case the outcome is (T, L) with positive probability. In the sequential case, it is never (T, L) and, thus, it is always inefficient.

It is interesting to contrast these observations with Bergemann and Välimäki [2]. They consider a dynamic common agency game, in which a set of principals face the same agent repeatedly. At each time, principals make offers to the agent contingent on the action that the agent chooses at the present time. The agent's and principals' payoff functions depend on time, the current action chosen by the agent, and all the previous actions chosen by the agent. Thus, in making their offers, principals must take into account the future effect of the current action chosen by the agent. Similarly, in choosing the current action, the agent does not only take into account current contributions and current direct payoff, but he must also consider the future effect of the current action. Bergemann and Välimäki prove that all Markov-perfect equilibria in truthful strategies of the dynamic common agency game are efficient.

The difference between Bergemann and Välimäki's dynamic common agency and an agent-sequential transfer game is that in their model there is a unique agent. Let us look at a modification of the example above that fits Bergemann and Välimäki's framework. Suppose that the payoff is as above. The game is played in an agent-sequential manner. However, we suppose that Agent 1 and Agent 2 are the same person, so that there is a unique agent who maximizes the undiscounted sum of the transfers received at time 1 and at time 2.

Bergemann and Välimäki predict that this game has a truthful equilibrium that induces the efficient outcome and indeed we can construct such equilibrium. If the agent chooses T in time 1, then he gets a transfer of 2 at time 2. If the agent chooses B at time 1, then he gets a payoff from transfers of 0 at time 2. At time 1, Principal 2 must offer at least 2 more than Principal 1 in order to induce the agent to choose B. There exists a subgame perfect equilibrium in which $t_T^{11} = 0$, $t_B^{21} = 2$, $t_L^{12} = t_R^{22} = 2$, and the agent chooses (T, L).

9 Conclusions

The main lesson of this analysis is that in games played by agents there may exist a type of inefficiency that is absent when either there is only one agent (Bernheim and Whinston [3]) or there is only one principal (Segal [12]). We may call it a *strategic inefficiency*. We have identified a condition in terms of game balancedness for this inefficiency to exist .

In the three examples provided in the Introduction, the presence of a strategic inefficiency is important for policy purposes. In lobbying, common agency tells us that if all citizens are represented by lobbies, all lobbies can make contributions, and there are no transaction costs, then the outcome of a 'pure corruption' model is always efficient (in a utilitarian sense). Our analysis says that this result is not true anymore if we take into account the multiplicity of policy-makers.

In supply contracts, if there is only one retailer, unrestricted vertical contracting leads to optimal allocations (Bernheim and Whinston [4, Proposition 1]). This result supports the view that a firm cannot profitably use exclusive dealings to foreclose a rival. However, our analysis suggets that that result does not extend to the case with multiple retailers, in which, even with unrestricted bilateral vertical contracting, the equilibrium may be inefficient. Indeed, in Subsection 4.2 we showed that the Opposite Interest Game, which has no efficient equilibrium, may be reinterpreted as a vertical supply problem.

In auctions, Subsection 8.2 has shown that a sequence of efficient auctions (in the sense that each auction assigns the object to the bidder with the highest valuation - which includes the bidder's strategic considerations) is not guaranteed to be efficient. Hence, a governement who - say - privatizes some assets through several rounds of auctions, should be aware of the possibility of strategic inefficiencies.

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